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MATHEMATICAL MODELLING OF THE MICROWAVE-DIAGNOSTICS OF IMPEDANCE SURFACES FOR THE REFLECTED SIGNAL OF UNKNOWN PHASE

M. S. Soppa

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A computer system of electromagnetic sounding data processing which allows the retrieval of the impedance distribution function of a cylindrical surface is considered. Calculation results are presented.

Key words: *mathematical modeling, inverse scattering problems, integral equations, signal phase retrieval.*

An important problem of science and engineering is to study methods of microwave diagnostics and remote sounding by electromagnetic waves of various frequency and intensity [1, 2]. In this direction, modern approaches of computational mathematics have been actively developed and specialized software and hardware have been designed. Mathematical modeling in this area involves numerical solution of inverse problems of diffraction in various formulations. We note approaches to the solution of inverse problems which use series expansions [3], analytical functions [4], exact or asymptotic solutions [5], and direct methods of residue minimization of the type of coordinate descent, which under the conditions of high ravineness of the objective function leads to rather tedious computations [6].

In the present paper, we propose an approach which uses a modified impedance boundary condition of the Leontovich type condition [7]. In this case, retrieving the impedance from the total complex-valued scattered field, the initial inverse problem in the differential formulation can be reduced to a linear integral-operator equation which admits effective discretization and regularization.

We consider the numerical solution of inverse problems of electromagnetic-wave diffraction for the case where the phase function of the scattering diagram is considered unknown (only the reflected-field modulus is known). It should be noted that observation results are much easier to obtain if the modulus rather than the total complex-valued magnitude of the reflected field is measured. The measurement of the reflected-wave phase is rather labor-consuming and requires accurate synchronization of transmit-receive devices at various points in space and very exact determination of the coordinates of these points. The reflecting surface properties are described by impedance boundary conditions. The employed mathematical model includes the two-dimensional Helmholtz equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + k^2 u = 0, \qquad (x, y) \in D$$

with the boundary condition

$$\frac{\partial u(x, y)}{\partial n} - i\xi u_0(x, y) = 0, \qquad (x, y) \in S.$$

Here $u = H_z(x, y)$ is the nonzero component of the electromagnetic field, $k = 2\pi/\lambda$ is the wavenumber, λ is the wavelength, $\xi = kW/W_0$ (W is the surface impedance which describes the processes in the surface layer of the

Novosibirsk State University of Architecture and Civil Engineering, Novosibirsk 630008; soppa@ngs.ru.
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conductor during interaction with the electromagnetic field and $W_0 = 120\pi$ is the wave resistance of the free space), and u_0 is the solution in the case of an ideally conducting surface S.

To retrieve the boundary impedance function W, one also needs data obtained by measuring the modulus of the scattered electromagnetic field $|h_g(M)|$ (specified values) at a finite set of points of the far field $M \in \{M_i, i = 1, \dots, m\}$. This implies that the required function W should give a minimum for the functional

$$J = \sum_{i=1}^{m} ||h(M_i)| - |h_g(M_i)||^2.$$

The inverse problem is solved by transformation to integral equations. For the case of an unknown phase of the reflected signal, we obtain the following nonlinear integral-operator equation to retrieve the impedance distribution for a fixed shape of the object:

$$\Big|\frac{k\gamma}{2\pi W_0}\int\limits_S \Big(ig(M,P) - \frac{\partial g(M,P)}{\partial n_P}\,A^{-1}B\Big)u_0 W\,dS_P + \frac{\gamma}{2\pi}\int\limits_S \frac{\partial g(M,P)}{\partial n_P}\,u_0\,dS_P\Big| = |h_g(M)|. \qquad (1)$$

Here $g = i\pi H_0^{(1)}/2$ is the fundamental solution of the Helmholtz equation, $H_0^{(1)}$ is the zero-order Hankel function of the first kind, and A and B are operators of solution of the direct problem:

$$A\eta = \frac{1}{2}\,\eta + \frac{1}{2\pi}\int\limits_S \eta\,\frac{\partial g}{\partial n}\,dS, \qquad B\eta = \frac{i}{2\pi}\int\limits_S \eta g\,dS.$$

We consider the unknown phase function of the scattered field $\theta(M)$. Multiplication of both sides of the equation by $e^{i\theta}$ yields relation (1), whose left and which right sides do not contain moduli.

The problem is discretized using the boundary-element method: the contour is replaced with a closed broken line consisting of n segments — panels. At control points (collocation points) on all panels, it is necessary to require that the boundary condition be satisfied. The values of all required functions are considered constant within each panel. A few neighboring panels can be united into segments, whose surface properties are known *a priori*. Integrals of the Hankel function and its derivatives over the panels are calculated using the rectangle formula with an adaptive choice of the number of nodes.

We assume that the parameters m and n are linked by the relation $m = 2n$. We introduce the additional variables $\mathrm{Re}\,h_g$ and $\mathrm{Im}\,h_g$. Then, the required parameters are $3n$ complex-valued parameters: the vector $\boldsymbol{W}$, whose components are functions of the impedance distribution on the surface of the object W_i $(i = \overline{1,n})$ and the vector $\boldsymbol{h}_g = (\boldsymbol{h}_{g1}, \boldsymbol{h}_{g2})$, whose components are the measured values of the scattered field divided into two groups: h_{g1i} $(i = \overline{1,n})$ and h_{g2i} $(i = \overline{n+1, 2n})$.

Discretization of the integral-operator equation (1) multiplied by $e^{i\theta}$ leads to the system of $2n$ complex linear equations

$$C_1\boldsymbol{W} = \boldsymbol{h}_{g1} + \boldsymbol{f}_1, \qquad C_2\boldsymbol{W} = \boldsymbol{h}_{g2} + \boldsymbol{f}_2,$$

where C_1 and C_2 are square matrices of order n. We express the surface impedance vector from the first group of equations and substitute it into the second group of equations:

$$C_2C_1^{-1}\boldsymbol{h}_{g1} = \boldsymbol{h}_{g2} + \boldsymbol{f}_2 - C_1^{-1}\boldsymbol{f}_1.$$

This system of equations with complex coefficients is represented as a real one:

$$A_1\boldsymbol{h}_1 + A_2\boldsymbol{h}_2 = \boldsymbol{f}_3.$$

Here

$$A_1 = \begin{pmatrix} -\mathrm{Im}\,C_2C_1^{-1} & -E \\ \mathrm{Re}\,C_2C_1^{-1} & 0 \end{pmatrix}, \qquad A_2 = \begin{pmatrix} \mathrm{Re}\,C_2C_1^{-1} & 0 \\ \mathrm{Im}\,C_2C_1^{-1} & -E \end{pmatrix},$$

$$\boldsymbol{h}_1 = (\mathrm{Im}\,\boldsymbol{h}_{g1}, \mathrm{Re}\,\boldsymbol{h}_{g2}), \quad \boldsymbol{h}_2 = (\mathrm{Re}\,\boldsymbol{h}_{g1}, \mathrm{Im}\,\boldsymbol{h}_{g2}), \quad \boldsymbol{f}_3 = (\mathrm{Re}\,(\boldsymbol{f}_2 - C_1^{-1}\boldsymbol{f}_1), \mathrm{Im}\,(\boldsymbol{f}_2 - C_1^{-1}\boldsymbol{f}_1)).$$

From this we obtain $\boldsymbol{h}_1$:

$$\boldsymbol{h}_1 = -A_1^{-1}A_2\boldsymbol{h}_2 + A_1^{-1}\boldsymbol{f}_3. \qquad (2)$$

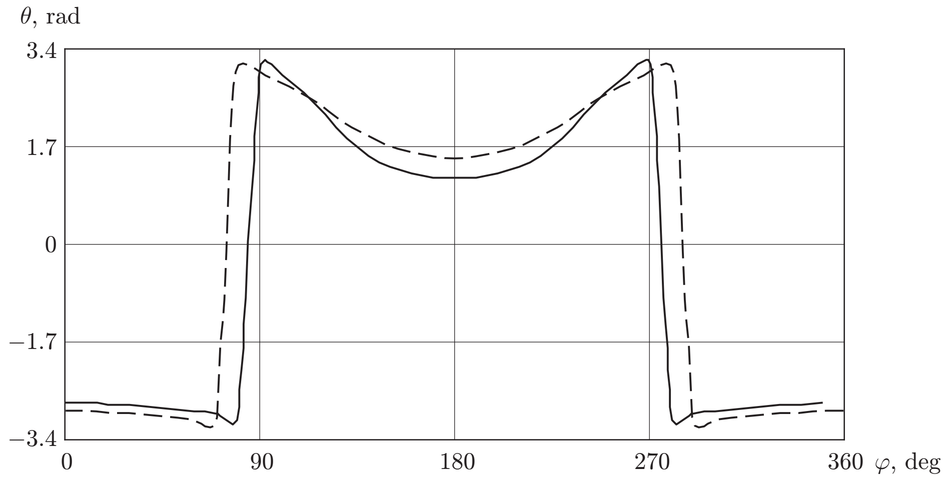

Fig. 1. Reflected-signal phase versus the azimuthal angle φ: the solid curve is the retrieved phase function; and the dashed curve is the phase function of the scattered wave in the case of a perfectly conducting scatterer.

The imaginary and real parts of the scattered-field values are linked by the relations

$$\operatorname{Im} h_{g1i} = \mu_i \sqrt{|h_{gi}|^2 - (\operatorname{Re} h_{g1i})^2}, \qquad i = \overline{1,n},$$

$$\operatorname{Re} h_{g2i} = \mu_i \sqrt{|h_{gi}|^2 - (\operatorname{Im} h_{g2i})^2}, \qquad i = \overline{n+1,2n}$$

(μ_i are the beforehand unknown signs of the imaginary and real parts). Substitution of representation (2) into these relations yields the following system of nonlinear equations of special form for the vector $\boldsymbol{v} = \boldsymbol{h}_2 = (\operatorname{Re} \boldsymbol{h}_{g1}, \operatorname{Im} \boldsymbol{h}_{g2})$:

$$G\boldsymbol{v} + \boldsymbol{b} = \boldsymbol{f}. \tag{3}$$

Here $f_i = \mu_i \sqrt{|h_{gi}|^2 - v_i^2}$, $i = \overline{1,m}$. The square matrix $G = -A_1^{-1} A_2$ of order m and the vector $\boldsymbol{b} = A_1^{-1} \boldsymbol{f}_3$ have known components. System (3) can have a nonunique solution due to nonlinearity. The uniqueness and physical interpretation of the solution is ensured by using the a priori information on it, which determines the choice of the parameters μ_i ($i = \overline{1,m}$). Additional regularization is provided by seeking a solution in the compact set $\boldsymbol{v} \in \{|v_i| \leqslant |h_{gi}|,\ i = \overline{1,m}\}$. The matrix G has the important property that, to construct the matrix inverse to it, it is sufficient to change signs at the elements constituting its diagonal blocks. Indeed, denoting the square blocks constituting the matrix G by $R = \operatorname{Re} C_2 C_1^{-1}$ and $I = \operatorname{Im} C_2 C_1^{-1}$, we obtain

$$G = \begin{pmatrix} -R^{-1}I & R^{-1} \\ R + IR^{-1}I & -IR^{-1} \end{pmatrix}, \qquad G^{-1} = \begin{pmatrix} R^{-1}I & R^{-1} \\ R + IR^{-1}I & IR^{-1} \end{pmatrix}.$$

Thus, the matrix G is inverted without loss of accuracy. To solve the constructed nonlinear system of special form (3), one can use an iterative method in which the kth approximation is found from the formula

$$\boldsymbol{v}^{(k)} = \boldsymbol{v}^{(k-1)} - \varepsilon(\boldsymbol{v}^{(k-1)} - G^{-1}(\boldsymbol{f} - \boldsymbol{b})), \qquad k = 1, 2, 3, \dots .$$

The iterative Newton method is implemented using the algorithm

$$\boldsymbol{v}^{(k)} = \boldsymbol{v}^{(k-1)} - \varepsilon(DF(\boldsymbol{v}^{(k-1)}))^{-1} F(\boldsymbol{v}^{(k-1)}), \qquad k = 1, 2, 3, \dots ,$$

where $F(\boldsymbol{v}^{(k-1)}) = G\boldsymbol{v}^{(k-1)} + \boldsymbol{b} - \boldsymbol{f}(\boldsymbol{v}^{(k-1)})$; the elements of the differential DF have the explicit representation:

$$DF(\boldsymbol{v}^{(k-1)})_{ij} = G_{ij} + \delta_{ij}\mu_i \frac{v_i^{(k-1)}}{\sqrt{|h_{gi}|^2 - \left(v_i^{(k-1)}\right)^2}},$$

δ_{ij} ($i = \overline{1,m}$; $j = \overline{1,m}$) is the Kronecker symbol; this method has a high rate of convergence for a value of the small parameter $\varepsilon \approx 0.001$. The initial approximation should satisfy the natural condition

$$v_i^{(0)} \in [-|h_{gi}|, |h_{gi}|].$$

After finding the real and imaginary parts of the field at the points of measurements and the surface impedance distribution, it is possible to calculate the scattered field $u^s = h$, and, then, using the relation

$$\theta(M) = \arctan \frac{\operatorname{Im}(u^s(M))}{\operatorname{Re}(u^s(M))}$$

to determine the phase function of the scattered field at any point in space.

The problem of retrieving the impedance distribution on a circular cylinder (in the case of H-waves) with a cross-sectional diameter $d = \lambda/\pi$ (see Fig. 1) was considered as an example.

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